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Adding these equations, we get

$$nu = \int_0^\infty [f(x) - f(k^n x)] \frac{dx}{x} = \int_0^\infty [f(x) - f(rx)] \frac{dx}{x},$$

where $r = k^n$.

Now, suppose n to become infinite, r remaining constant, then k will approach unity as a limit, and we have

$$\int_0^\infty [f(x) - f(rx)] \frac{dx}{x} = n \int_0^\infty [f(x) - f(r^{1/n} x)] \frac{dx}{x}.$$

The right member of this equation (in the limit) reduces to the indeterminate form 0/0. Differentiating in this form with respect to $1/n$, we get

$$- \int_0^\infty f'(x) \log r dx = \log r[f(0) - f(\infty)].$$

Now in the given integral, $f(x) = (\tan^{-1} ax)^2$. Hence,

$$f(0) = 0, \qquad f(\infty) = \tfrac{1}{4}\pi^2;$$

also,

$$r = \frac{b}{a}.$$

Hence the integral is equal to $\frac{1}{4}\pi^2$ (log a — log b).

This problem may be readily solved by Frullani's Theorem, Williamson's *Integral Calculus*, page 155. Professor J. Scheffer solved the problem by means of this Theorem, but failed to notice that in the above problem $\phi(ax) = [\tan^{-1} ax]^2$ instead of $\tan^{-1} ax$. Ed. F.

327. Proposed by RICHARD P. LOCHNER, Philadelphia, Pa.

A hound is at the middle point of the side of a square field and a fox is at an adjacent corner. How far will the hound run to catch the fox if the fox runs on the perimeter of the field and the hound runs directly towards the fox at all times, the hound running n times as fast as the fox. Where will the race end?

No solution of this problem has been received.

328. Proposed by M. E. GRABER, Tiffin, Ohio.

Prove that

$$\frac{E_m}{\pi/2m} \int_0^{\pi/2m} \left[\sin a + \sin\left(\frac{\pi}{m} + a\right) + \cdots + \sin\left(\pi \frac{m-1}{m} + a\right) da \right] = \frac{2mE_m}{\pi}.$$

SOLUTION BY A. M. HARDING, University of Arkansas.

From trigonometry we have

$$\sin a + \sin\left(\frac{\pi}{m} + a\right) + \cdots + \sin\left(\pi \frac{m-1}{m} + a\right) = \frac{\sin\left(a + \dfrac{m-1}{2m} \cdot \pi\right)}{\sin \dfrac{\pi}{2m}}.$$

Hence the given integral takes the form

$$\frac{E_m}{\frac{\pi}{2m}}\cdot\frac{1}{\sin\frac{\pi}{2m}}\int_0^{\pi/2m}\sin\left(a+\frac{m-1}{2m}\cdot\pi\right)da$$

$$=\frac{2mE_m}{\pi}\cdot\frac{1}{\sin\frac{\pi}{2m}}\cdot\left[-\cos\left(a+\frac{m-1}{2m}\pi\right)\right]_0^{\pi/2m}$$

$$=\frac{2mE_m}{\pi}\cdot\frac{1}{\sin\frac{\pi}{2m}}\cdot\sin\frac{\pi}{2m}=\frac{2mE_m}{\pi}.$$

Solved similarly by Geo. W. Hartwell and J. Scheffer.

329. Proposed by C. N. SCHMALL, New York City.

Show that the general differential equation

$$\frac{d^ny}{dx^n}+P_1\frac{d^{n-1}y}{dx^{n-1}}+P^2\frac{d^{n-2}y}{dx^{n-2}}+\cdots+P_{n-1}\frac{dy}{dx}+P_ny=V,$$

where P_1, P_2, $\cdots$ P_n, V are known functions of x, has a solution in the form

$$y-v_1\int v_2dx\int v_3dx\cdots\int\frac{Vdx}{v_1v_2v_3\cdots v_n}.$$

Solution by A. M. Harding, University of Arkansas.

Let $y = v_1z_1$. Then the given differential equation becomes

$$v_1\frac{d^nz_1}{dx^n}+Q_1\frac{d^{n-1}z_1}{dx^{n-1}}+\cdots+Q_{n-1}\frac{dz_1}{dx}$$
$$+z_1\left[\frac{d^nv_1}{dx^n}+P_1\frac{d^{n-1}v_1}{dx^{n-1}}+\cdots+P_{n-1}\frac{dv_1}{dx}+P_nv_1\right]=V. \quad (1)$$

Now suppose the expression in brackets, when equated to zero, is satisfied by v_1. In that case this expression will vanish, and we have left a linear equation of the $(n-1)$th degree in dz_1/dx.

Putting then

$$\frac{dz_1}{dx}=y_1, \quad (2)$$

and $Q_r = v_1R_r$, we have

$$\frac{d^{n-1}y_1}{dx^{n-1}}+R_1\frac{d^{n-2}y_1}{dx^{n-2}}+\cdots+R_{n-1}y_1=\frac{V}{v_1}.$$

Now this equation has the same form as the original equation and is one degree lower. Proceeding in the same way we at last get

$$y_n=\frac{V}{v_1v_2v_3\cdots v_{n-1}v_n},$$